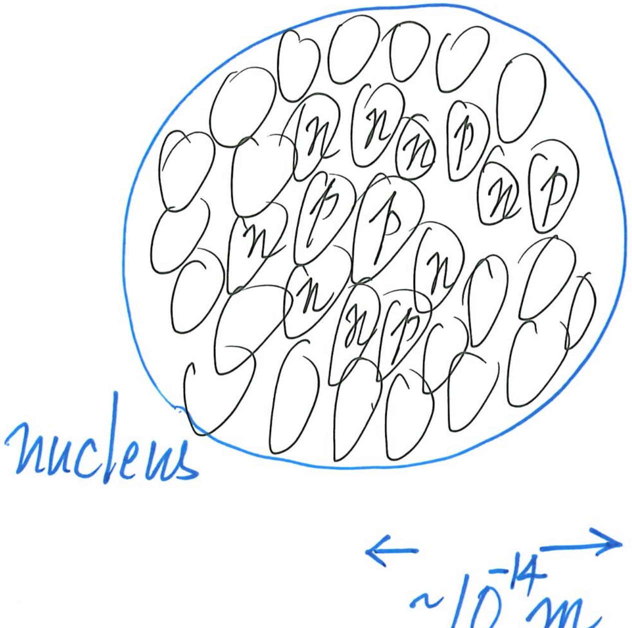
Tunneling and its Applications

- Traveling Waves in QM : Necessity and issues
- Continuity Equation and Probability Current density
- Tunneling
- Applications

Motivation (Phenomena)

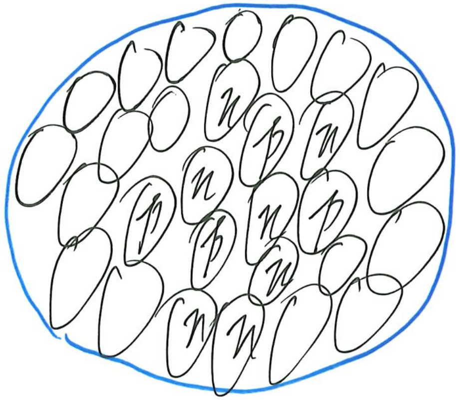
- α - decay

neutrons + protons

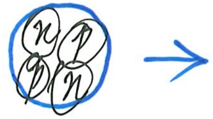


nucleons (p and n) bind by nuclear force

only effective when nucleons are very close (~10^{-15} m)

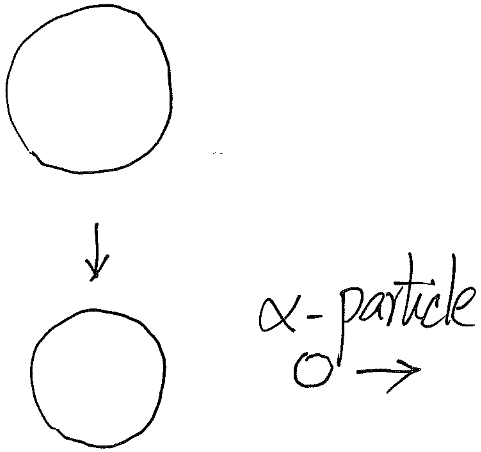


α - particle

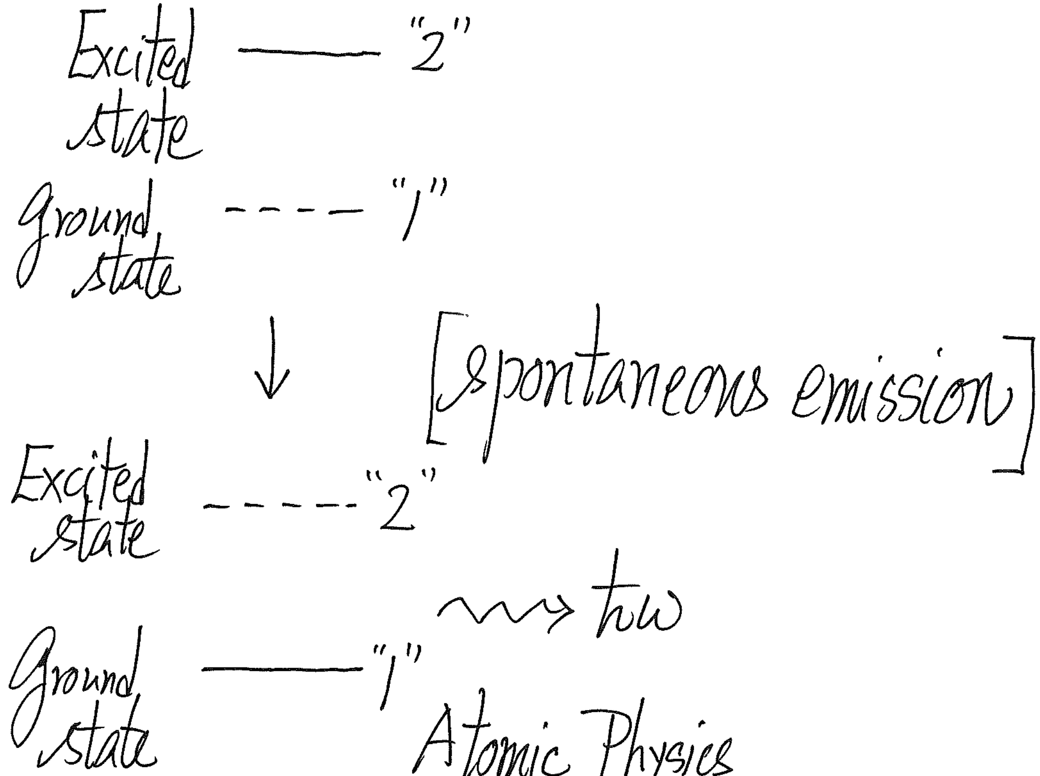


[helium nucleus]
(2 protons + 2 neutrons)

Analogy



Nuclear Physics



Atomic Physics

$$\frac{dN(t)}{dt} = -\lambda N \leftarrow \text{(probabilistic)} \rightarrow \frac{dN_2(t)}{dt} = -\lambda_{21} N_2(t)$$

$N(t)$ = # nuclei not decayed at time t ("in excited state")

$\therefore$ Life time $\tau = \frac{1}{\lambda}$

half-life $t_{1/2} \equiv \tau \cdot \ln 2$

Data of 5 α-particle emitting nuclei

Nucleus	k.e. (MeV)	half-life $t_{1/2}$
^{216}Ra	9.5	0.18 μs
^{194}Po	7.0	0.7 s
^{240}Cm	6.4	27 days
^{226}Ra	4.9	1600 years
^{232}Th	4.1	14 billion years

α-particle k.e. don't vary much!

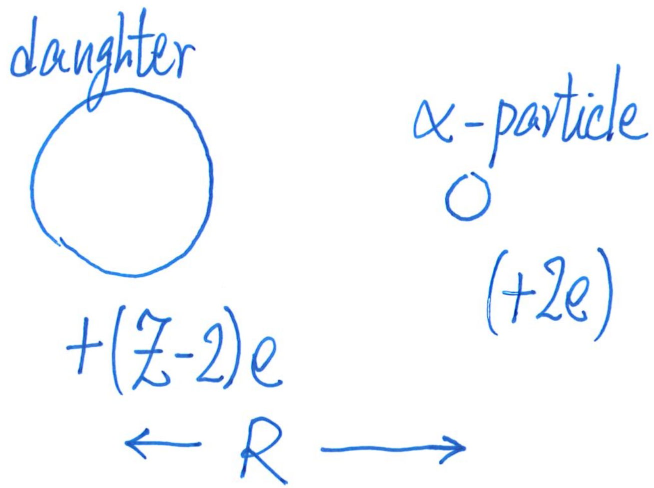
$t_{1/2}$ vary by a lot!

Many orders of magnitude

Note correlation:

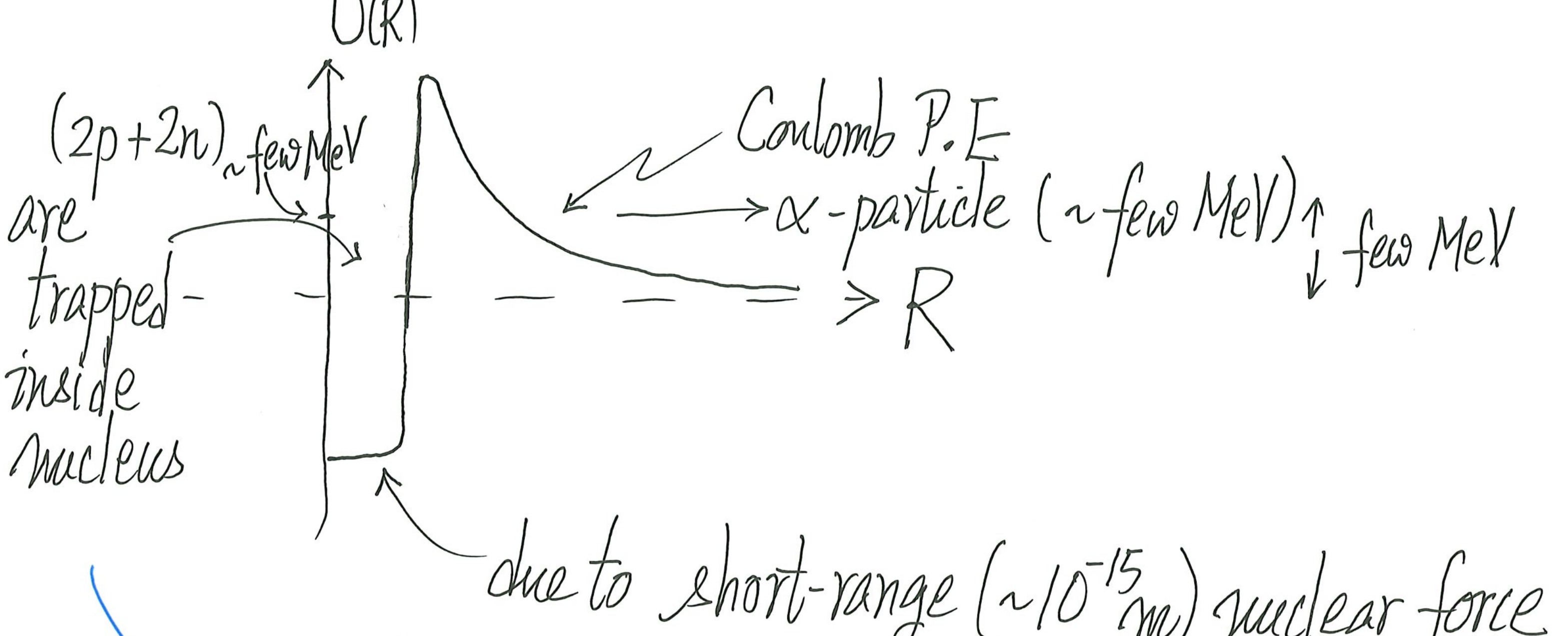
longer $t_{1/2}$ $\leftrightarrow$ smaller α-particle k.e.

shorter $t_{1/2}$ $\leftrightarrow$ higher α-particle k.e.



Coulomb repulsion

$$\left[P.E. = + \frac{(Z-2)\cdot 2e^2}{4\pi\varepsilon_0 R} \right]$$



Q: Does α-particle climb out of the barrier?
[No! α-particle k.e. is NOT that high!]

Q: How does α-decay occur? [Tunneling]

To understand tunneling properly†, we need some formal QM on:

- Traveling waves
- Continuity Equation and Probability Current Density

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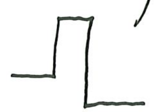
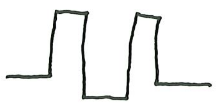
† Some books introduced tunneling in a way that students only know how to do  . Here, we will know how to do  ,  ,  , etc. at the end.